

Low Bond Yields and Safe Portfolio Withdrawal Rates

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Executive Summary

- ▶ Yields on government bonds are well below historical averages. These low yields will have a significant impact for retirees, who tend to invest heavily in bonds, because portfolio returns in the earliest years of retirement have a larger impact on the likelihood that a retirement income strategy will succeed than returns later in retirement; this is known as sequence risk.
- ▶ The majority of research on sustainable withdrawal strategies has used a stochastic (Monte Carlo) simulation process based on long-term averages, where the expected return of an asset class is the same for each year of the simulation. While this approach is reasonable when markets are near long-term averages, we believe it is less useful when there is a significant and sustained deviation such as the current low bond yield market.
- ▶ In this paper we introduce a model that takes into account current bond yields and allows them to “drift” toward a higher value during retirement using an autoregressive model based primarily on historical relationships between asset classes. This approach can better replicate the actual bond returns a current or near retiree can expect during retirement both now and in the future.
- ▶ Using this model, we find a significant reduction in “safe” initial withdrawal rates, with a 4% initial real withdrawal rate having approximately a 50% probability of success over a 30-year period.
- ▶ We find a retiree who wants a 90% probability of achieving a retirement income goal with a 30-year time horizon and a 40% equity portfolio would only have an initial withdrawal rate of 2.8%. Such a low withdrawal rate would require 42.9% more savings if the retiree wanted to pull the same dollar value out of the portfolio annually as he or she would get with a 4% withdrawal rate from a smaller portfolio.

Low Bond Yields and Safe Portfolio Withdrawal Rates

Bond yields today are well below historical averages. This has significant implications because portfolio returns in the earliest years of retirement have a larger impact on the likelihood that a retirement income strategy will succeed than returns later in retirement. The majority of research on sustainable withdrawal strategies has used a stochastic (Monte Carlo) simulation process based on long-term averages, where the expected return of an asset class is the same for each year of the simulation. While this approach is reasonable when markets are near long-term averages, we believe it is less useful when there is a significant and sustained deviation such as the current low bond yield market.

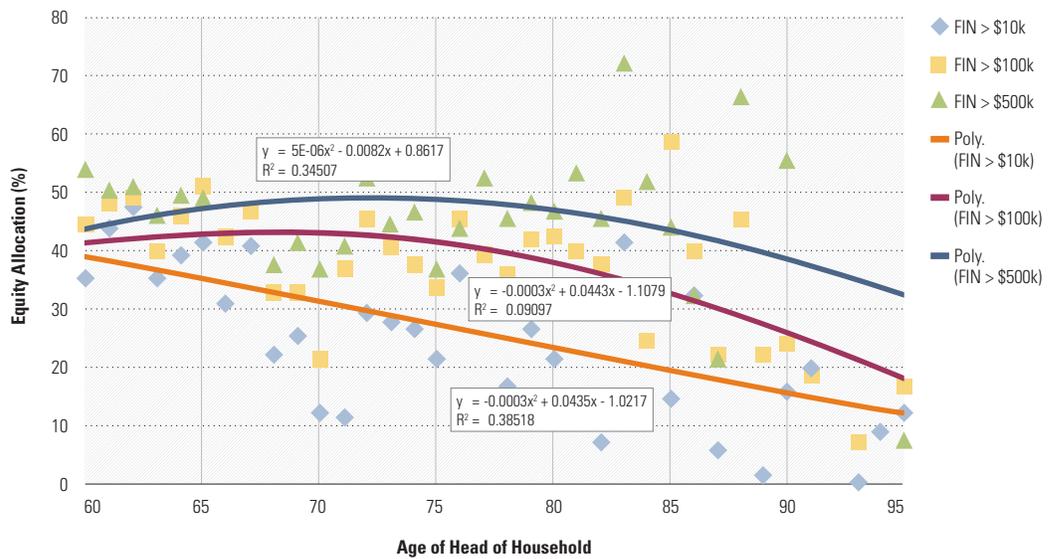
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Bond Yields Today

These are trying times for bond investors. The yield on 10-year government bonds is approximately 1.8% and the yield for the High Quality Market Corporate Bond Yield Curve at 10 years is approximately 3.2%. These are both considerably below long-term averages.

Low bond yields have important implications for different types of investors, especially older investors, who tend to invest more conservatively than younger investors. This concept is depicted visually in Figure 1, which includes the median equity allocation for household’s financial assets (FIN), given different asset levels and ages.

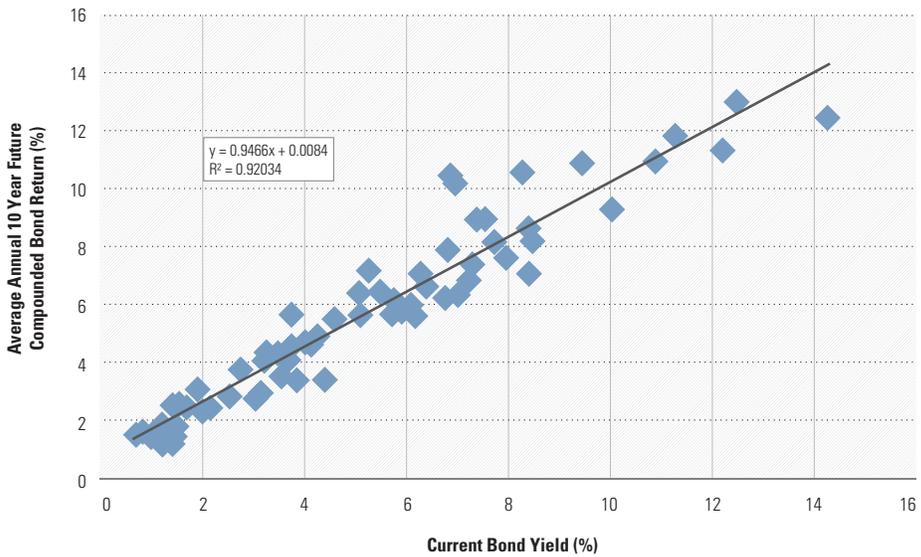
Figure 1: Median Equity Allocations of Financial Assets by Age for Various Levels of Financial Assets



Source: 2010 Survey of Consumer Finances

A high allocation to low-yielding bonds limits a retiree’s ability to generate income from retirement wealth. Unfortunately for today’s retiree, there is a very strong historical relationship between bond yields and the future returns realized by bond investors, even over prolonged periods. Figure 2 demonstrates the relationship between bond yields and the future average annualized total return of bonds using the Ibbotson Intermediate-Term Bond Index.

Figure 2: Relationship Between Bond Yields and Future Average Annualized 10 Year Bond Return



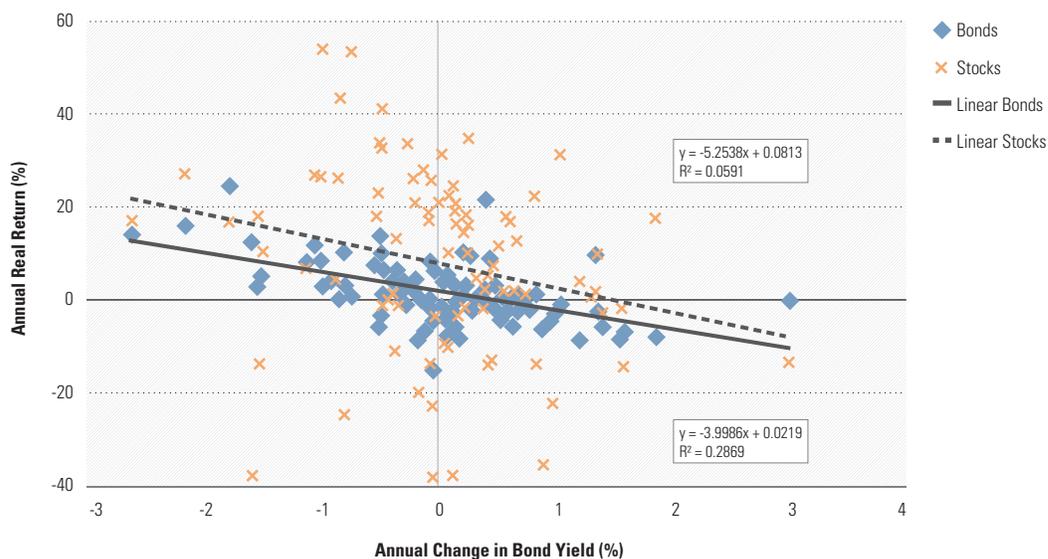
The historical relationship between bond yields in one period and the future average annualized total return of bonds has been quite strong, with a coefficient of determination (R^2) of 92.03%. This means that the current yield on bonds can describe 92.03% of the average annual 10-year future compounded bond total return. If we assume a current bond yield of 1% (which is slightly higher than the yield on the Ibbotson

Intermediate-term Government Bond Index as of December 2012), the average annualized bond total return over the next 10 years is expected to be 1.8% using the linear regression model in Figure 2. This return is almost 4% less than the 5.5% average annual return on the Ibbotson Intermediate-Term Government Bond Index from 1930 to 2011.

While rising bond yields would result in higher returns for new bond investors, it would negatively affect those currently holding bonds as the values of their low-yield bonds decline. One method to approximate the impact of a change in interest rates on the price of bonds is to multiply the bond's duration by the change in interest rates times negative one. For example, if interest rates increase by 2%, a bond with a duration of five years (the approximate current duration of the Barclays Aggregate Bond Index) would decrease by 10%. The impact on bonds with longer durations would be even more extreme.

While there is a negative relationship between bond yield changes and bond returns, there is also a slight negative relationship between the change in bond yields and the return on stocks. Figure 3 includes the real (inflation-adjusted) annual stock and bond returns for different historical changes in interest rates. While the R^2 between changes in bond yields and the subsequent real return on stocks has been relatively small (5.91%), there is still a negative relationship. This negative relationship implies stock returns may be lower in the foreseeable future as well.

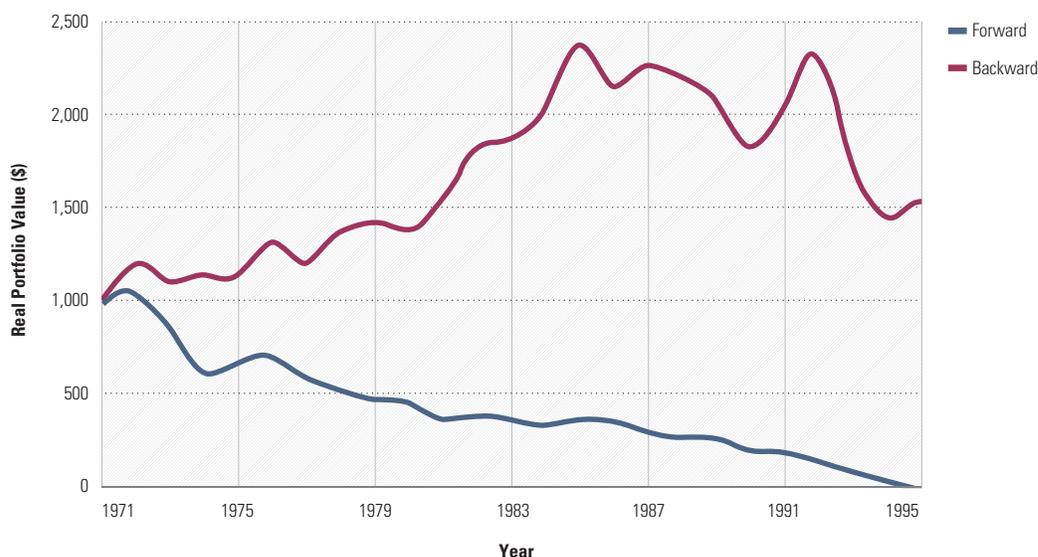
Figure 3: Real Stock and Bond Returns in Different Bond Yield Change Environments



Why Return Sequence Matters

Retirement income portfolios are sensitive to poor portfolio returns early in retirement - a concept known as sequence risk. Figure 4 illustrates differences in the real growth of a portfolio with an initial balance of \$1,000, assuming a 5% initial withdrawal rate when early returns are favorable and unfavorable.

Figure 4: Sequence Risk



For the Forward scenario in Figure 4, the individual is assumed to retire on December 31, 1971. By the end of the 24th year (December 31, 1995), the portfolio would be exhausted and no longer able to fund the income goal of the retiree. If, however, we use the same real returns but assume the retiree would experience them in reverse (the Backward scenario), the portfolio would still have a healthy balance (in fact, higher than the initial investment) by the end of the 24th year. Although the average annual return of the simulations is identical, the outcomes are very different.

The vast majority of models used to determine sustainable initial withdrawal rates from a portfolio use a single set of long-term values (e.g., returns, standard deviation, correlations) for the entire simulation. The values are either historical (e.g., based on a set of indices) or forward looking. The obvious problem with this approach is that it assumes the returns an investor is able to achieve are equally likely over the entire retirement period. In other words, the returns from one period to another do not show momentum or serial correlation. While this may generally be the case for equities, it is not the case with fixed income securities.

While the average annual arithmetic return on the Ibbotson Intermediate-Term Government Bond Index from 1930 to 2011 was 5.5%, the interest rate available on an intermediate-term government bond today is closer to 2.0%, which is 3.5% lower than the historic average. An analysis that assumes an average bond return of 5.5% in the first year (with a 6.5% standard deviation) will significantly overestimate early retirement portfolio returns.

The approach taken in this paper assumes that bond yields will eventually revert toward their long-term averages, but reflects the actual yields available to investors today for the first years of the simulation. This makes the analysis somewhat time sensitive (because bond yields are always changing), but provides a better estimate of the likely future bond returns available to investors within the context of interest rates today.

Research on Sustainable Withdrawal Rates

For most practitioners, the methodology for generating a safe retirement income from an investment portfolio was first addressed in Bengen (1994). Like most of the so-called safe withdrawal rate research that has fol-

lowed, Bengen uses an outcomes metric similar to Roy's (1952) Safety-First rule, which uses the probability of achieving a goal over some time period to define the optimal withdrawal amount from a portfolio.

The term "initial withdrawal rate" is commonly used within portfolio withdrawal strategy research to describe the initial percentage withdrawn from the portfolio. This withdrawal amount is assumed to increase thereafter by inflation. For example, an initial portfolio with a \$1 million balance and 4 percent initial withdrawal would yield \$40,000 in the first year. If inflation was 3 percent during the first year, the withdrawal for the second year would be \$41,200 ($\$40,000 * 1.03 = \$41,200$). This inflation adjusted withdrawal rate is deducted from the portfolio balance until retirement assets are exhausted. This represents a static approach to portfolio income because the income amount is determined initially (upon retirement) and is not revisited.

Bengen (1994) recommends a 4 percent initial withdrawal rate from a portfolio made up of 50 percent stocks and 50 percent intermediate-term treasuries, is sustainable for a minimum of 33 years for retirees age 60–65. Bengen (2006) later coined the term SAFEMAX to describe the maximum inflation-adjusted withdrawal rate that would allow for at least 30 years of withdrawals without exhausting one's savings during all of the rolling periods available in the historical data.

Later research by Cooley, Hubbard, and Waltz (1998), often called the Trinity study, generally confirmed Bengen's findings but increased the scope to different period lengths, initial withdrawal rates and types, and asset allocations. For example, Cooley, Hubbard, and Waltz (1998) note that if a retiree seeks a 75 percent probability of success, a 4-to-5 percent initial withdrawal would be a good place to start (assuming portfolios of 50 percent or more composed of large-company common stocks using historical data from 1926-1995). These findings have been affirmed by other research such as Milevsky, Ho, and Roberson (1997) and Jarrett and Stringfellow (2000), among others.

Asset allocation can significantly affect a portfolio's ability to sustain a given cash flow during retirement. Research by Ervin, Filer, and Smolira (2005), Tezel (2004), Cooley et al. (2003), and Kaplan (2005) demonstrate that portfolios with lower equity allocations tend to generate higher probabilities of ruin (in particular over retirement periods). These findings flow primarily from the use of historical market return averages in a Monte Carlo setting, since returns are generally assumed to be stationary and equities are assumed to have a higher return than bonds. Pfau (2011) used more forward-looking estimates when determining sustainable withdrawal rates and notes significant differences in the safety of different withdrawal rates.

Methodology

A model is constructed to generate returns for cash, bonds, and stocks, as well as inflation, that allows the expected yield on bonds to drift upward toward their historical average over time. The initial interest rate (seed value) is assumed to be 2.5%, based on interest rates today. This is the approximate yield on the Barclays Aggregate Bond Index as of January 1, 2013. Future bond yields are determined using an autoregressive model, detailed in Appendix 1, where the future bond yields are determined in reference to the historical bond yield while including a random error.

The returns experienced throughout the simulation are obviously going to be affected by the initial bond yield; however, the autoregressive model used for the analysis assumes that bond yields slowly revert back to their long-term averages. Figure 5 demonstrates this concept and includes the different percentile bond yields for years 0 through 40 within a given simulation. While the bond yield is the same (2.5%) in the beginning,

the median yield across scenarios drifts toward its long-run average as the simulations progress. The actual yields experienced within each year of each simulation are going to drift through time, but on average the model assumes that yields are stationary (i.e., mean reverting).

Figure 5: Model Yield Convergence

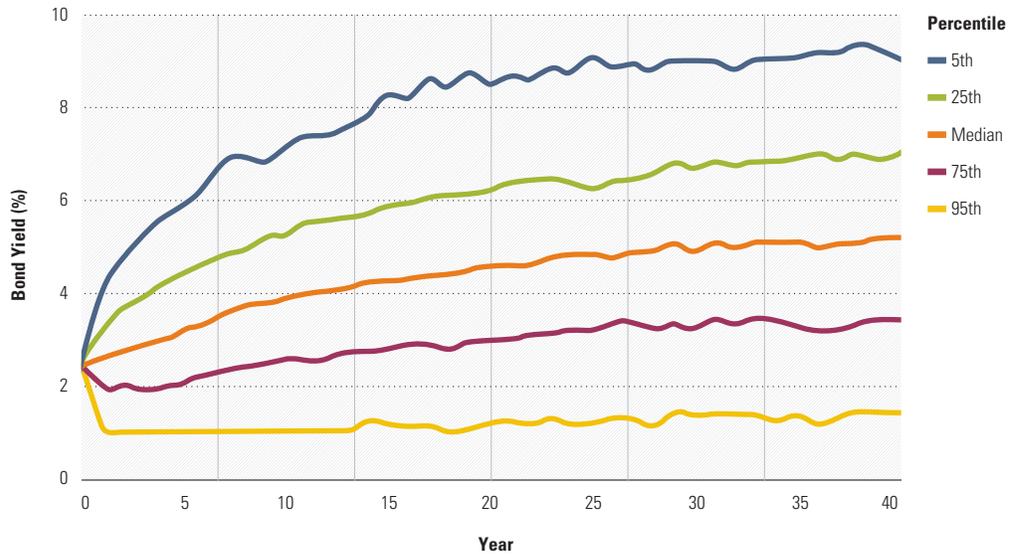


Table 1 includes the returns, standard deviation, and correlations for the different modeling components for the 30th year of a given simulation. These 30th year values can be viewed as the long-term assumptions of the model. We base our model primarily on the long-term annual returns, and relationships, between different asset classes. For our analysis we use 30-day Treasury Bills as the proxy for cash, the Ibbotson Intermediate-Term Government Bond Index as a proxy for Bonds, and the S&P 500 Index as a proxy for stocks. We make a few subjective adjustments to make the expected returns look more similar to Ibbotson’s 2012 long-term capital market forecasts. The most significant adjustment was a reduction in the assumed return for equities.

Table 1: Returns, Standard Deviations, and Correlations

	Cash TR (%)	Bond TR (%)	Bond Yld (%)	Stock TR (%)	Inflation (%)
Return	3.02	5.14	5.01	9.89	3.14
Standard Deviation	2.33	6.22	2.31	19.68	2.71
Correlations					
Cash TR	1.00	0.24	0.88	-0.02	0.45
Bond TR	0.24	1.00	0.29	0.11	-0.10
Bond Yld	0.88	0.29	1.00	0.00	0.28
Stock TR	-0.02	0.11	0.00	1.00	-0.08
Inflation	0.45	-0.10	0.28	-0.08	1.00

The historical returns on U.S. stocks have been considerably higher than the equity returns of other countries. For example, Dimson, Marsh, and Staunton find that the average annual inflation-adjusted geometric (i.e., compounded) return of U.S. stocks from 1900 to 2011 was 7.26%. In contrast, the

average annual real geometric return for equities in France was 3.06%, versus 4.24% for Switzerland, and 5.33% for the United Kingdom.

While it is impossible to determine whether or not equities in the United States will continue to outperform other countries, in order to be conservative, the forecasted equity return is reduced in the model. From January 1926 to December 2011, the average annual arithmetic return for the S&P 500 was 11.77%. This return is reduced by 2.0% so that the average expected arithmetic return for equities going forward is 9.77% (which is slightly different than the value in Table 1, because the Table 1 value is the average across the simulations). This was accomplished by reducing the intercept in the return model for equities.

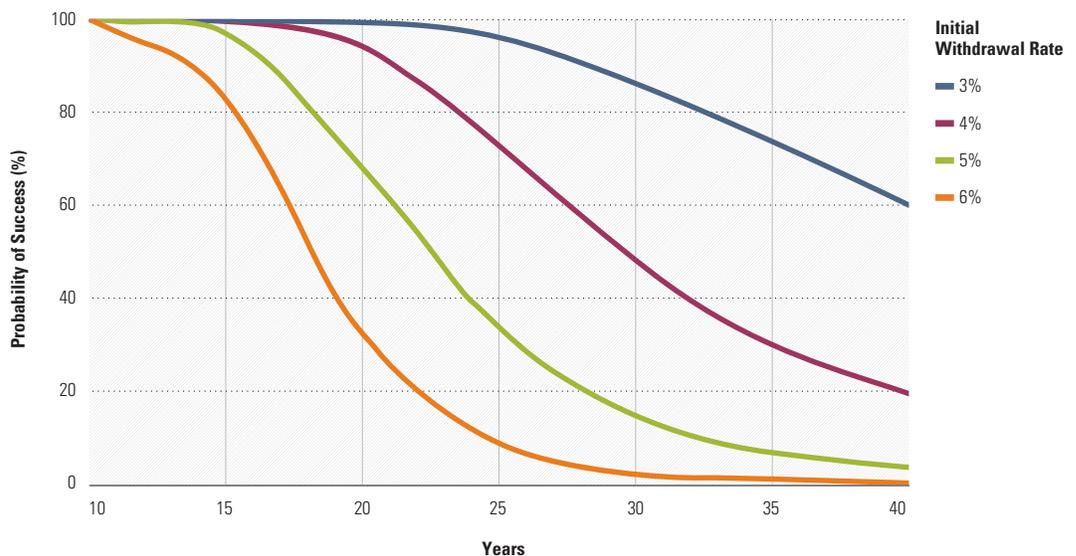
Each scenario in the analysis is based on a 10,000-run Monte Carlo simulation. Taxes and Required Minimum Distributions (RMDs) from the portfolio are ignored. The analysis assumes a 1.0% fee, or negative alpha, that is deducted from the portfolio value annually. This fee is included to account for unavoidable retirement portfolio expenses paid by the investor (e.g., mutual fund fees, advisor fees, account fees, etc.) for investment management.

The primary metric used to relay the risk of different initial withdrawal rates is the probability of success. The probability of success is the percentage of runs that are able to successfully achieve the target cash flow for the respective period. While the probability of success is an imperfect measure because it does not provide information about the magnitude of failure, it is the most prominent metric in withdrawal rate research and relied upon by advisors to illustrate the risk of a given withdrawal rate and retirement portfolio strategy.

Results

As noted in the Literature Review section, there is considerable research noting the safety of a 4% initial withdrawal rate. We find the relative safety of a 4% initial withdrawal rate may not be safe in a low yield environment. Figure 6 includes the probability of success for various initial withdrawal rates for a 40% equity portfolio from 10 to 40 years.

Figure 6: Probabilities of Success for Various Initial Withdrawal Rates for a 40% Equity Portfolio



We note the probability of success for a 4% initial withdrawal rate over a 30-year period for a 40% equity portfolio to be 48.2%, or slightly less than a coin flip. This is a considerably lower probability of success than what is noted in past research, which has tended to be above 80%. This result stems from three key differences in this study versus past studies (especially those that have used purely historical data). First, we use a model that incorporates the actual yields available to retirees today (that converges towards the long run expectation, on average). Second, we reduce the expected arithmetic return on equities by 2.0% (to 9.77%) to reflect a more realistic forecast for U.S. equities. Third, we assume a fee of 1.0% as a proxy for the asset management fees that are likely to be paid by an investor.

Given the results shown in Figure 6, an important question would be; what is a safe initial withdrawal rate given the assumptions of this model? This information is included in Table 2, which notes the initial withdrawal rates for various equity allocations, retirement periods, and probabilities of failure. For example, if a retiree with a 20% equity allocation wanted to plan for a 30-year time horizon and wanted a 90% probability of success, the initial withdrawal rate would be 2.7%.

Table 2: Initial Withdrawal Rates for Various Equity Allocations, Retirement Periods, and Probabilities of Success

		Retirement Period (Years)											
		15	20	25	30	35	40	15	20	25	30	35	40
Probability of Success (%)	99	20% Equity Allocation						40% Equity Allocation					
	95	5.0	3.6	2.8	2.2	1.9	1.6	4.6	3.3	2.5	2.1	1.8	1.6
	90	5.4	4.0	3.1	2.6	2.2	1.9	5.2	3.9	3.1	2.6	2.2	2.0
	80	5.7	4.2	3.3	2.7	2.3	2.1	5.6	4.2	3.4	2.8	2.5	2.2
	50	6.0	4.4	3.5	3.0	2.6	2.3	6.1	4.6	3.7	3.2	2.8	2.5
	50	6.6	5.0	4.1	3.4	3.0	2.7	7.0	5.5	4.5	3.9	3.5	3.2
Probability of Success (%)	99	60% Equity Allocation						80% Equity Allocation					
	95	3.9	2.8	2.2	1.9	1.5	1.3	3.4	2.3	1.8	1.4	1.2	1.1
	90	4.9	3.6	2.8	2.4	2.0	1.8	4.4	3.2	2.6	2.1	1.8	1.6
	80	5.4	4.0	3.2	2.7	2.4	2.2	5.1	3.8	3.0	2.6	2.2	2.0
	50	6.1	4.6	3.8	3.2	2.9	2.6	5.8	4.6	3.7	3.2	2.8	2.6
	50	7.4	5.9	4.9	4.3	3.9	3.6	7.8	6.2	5.3	4.6	4.2	3.9

Table 2 has important implications for both existing retirees and those approaching retirement. The reciprocal of the initial withdrawal rate is the amount an individual must have saved to achieve an income goal. For example, if a retiree wanted a 4% initial withdrawal rate, he or she would need a portfolio that was 25 times ($1 / 4.0\% = 25$) the annual income he or she desires in retirement. In contrast, if the initial withdrawal rate decreases to 3.0%, then the amount the retiree must save in order to withdraw the same annual dollar amount as in the 4.0% example increases to 33.33 times the target income amount. This may seem counterintuitive, but the larger the portfolio, the smaller the withdrawal rate required to provide the target annual income.

While the difference between a 3.0% initial withdrawal rate and a 5.0% initial withdrawal rate may not seem material, the 3.0% initial withdrawal rate requires 66.7% more savings than the 5.0% initial withdrawal rate to produce the same annual income. One way to reduce the required savings amount

would be to potentially take on more risk during retirement by increasing allocation to equities. Unfortunately, increasing portfolio risk does not have a material impact. For example, the initial withdrawal rate for a 20% equity portfolio with a 90% probability of success for a 30-year retirement period is 2.7%. If the retiree increased the equity portion of the portfolio to 60% and lowered the probability of success to 80%, he or she could only raise the initial withdrawal rate to 3.2%. This would require 18.5% less savings, but would subject the retiree to considerably more market risk, which is something that is not captured in the probability of success metric.

Conclusions

This paper introduced a model that takes into account current bond yields when determining the probability of success for different initial withdrawal rates over different time periods and for different equity allocations. Using a model that incorporates how bond yields are likely to move (or drift) through time is a better approach to modeling returns retirees are likely to experience than assuming the same average return for each year of the simulation as in previous studies. This is especially important because the order of returns experienced during retirement can significantly affect the likelihood of a retiree achieving his or her income goal (something known as sequence risk).

We find that a 4% initial withdrawal rate has approximately a 50% probability of success over a 30-year period. This success rate is much lower than past studies, which have typically noted a probability of success above 80%. This has significant implications on the likelihood of success for retirees today as well as how much those nearing retirement need to have saved to ensure a successful retirement. For example, a retiree who wants a 90% probability of achieving a retirement income goal with a 30-year time horizon and a 40% equity portfolio would only have an initial withdrawal rate of 2.8%. Such a low withdrawal rate would require 42.9% more savings if the retiree wanted to pull the same dollar value out of the portfolio annually as he or she would get with a 4% withdrawal rate from a smaller portfolio.

Source: Morningstar Investment Management

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Monte Carlo is an analytical method used to simulate random returns of uncertain variables to obtain a range of possible outcomes. Such probabilistic simulation does not analyze specific security holdings, but instead analyzes the identified asset classes. The simulation generated is not a guarantee or projection of future results, but rather, a tool to identify a range of potential outcomes that could potentially be realized. The Monte Carlo simulation is hypothetical in nature and for illustrative purposes only. Results noted may vary with each use and over time.

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Appendix

The first step in the return forecasting model is to create an interest rate (i.e., yield value) for each year of each run. The simulation begins with an initial interest rate (i.e., the seed value) which is set to 2.5%, which is the approximate yield to maturity of the Barclays Aggregate Bond Index on January 1, 2013. The yield for each subsequent year (Y_t) is based on equation 1, which is an autoregressive model with a single lag (the previous year's yield), where $\alpha_Y = .269\%$ and $\beta_Y = .949$, and ε_Y is an independent white noise that follows a standard normal distribution with a mean of 0% and a standard deviation of .90%. The yield is assumed to be a minimum of 1.0% and a maximum of 10.0%.

Equation 1

$$Y_t = \alpha_Y + \beta_Y Y_{t-1} + \varepsilon_Y$$

After the annual bond yields have been determined, the total return for cash is determined based on equation 2, where $\alpha_c = -2.024\%$, $\beta_c = .978$, Y_t = the yield for that year, $\beta_{y\Delta c} = .321$, $\Delta Y = Y_t - Y_{t-1}$, and ε_c is an independent white noise that follows a standard normal distribution with a mean of 0% and a standard deviation of 1.0%. The total return is assumed to be a minimum of 0% and a maximum of 10%.

Equation 2

$$r_c = \alpha_c + \beta_{yc} Y_t + \beta_{y\Delta c} \Delta Y + \varepsilon_c$$

The next step is to determine the return for bonds, stocks, and inflation, based on equation 3. The coefficient values equation 3 for bonds, stocks, and inflation are included in Table A1.

Equation 3

$$r_i = \alpha_i + \beta_{yi} Y_t + \beta_c r_{c,t} + \beta_{y\Delta i} \Delta Y + \varepsilon_i$$

Table A1: Coefficients for Bonds, Stocks and Inflation

	α_i	β_{yi}	β_c	$\beta_{y\Delta i}$	σ for ε_i (%)	Min (%)	Max (%)
Bonds	.920	.678	.446	-3.714	5.066	-15	40
Stocks	7.951	-.308	.593	-4.221	19.358	-100	200
Inflation	2.983	.964	-.554	1.012	2.088	-10	20